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FINANCIAL RATIOS, DISCRIMINANT ANALYSIS AND THE PREDICTION OF CORPORATE BANKRUPTCY

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ACADEMICIANS SEEM to be moving toward the elimination of ratio analysis as an analytical technique in assessing the performance of the business enterprise. Theorists downgrade arbitrary rules of thumb, such as company ratio comparisons, widely used by practitioners. Since attacks on the relevance of ratio analysis emanate from many esteemed members of the scholarly world, does this mean that ratio analysis is limited to the world of "nuts and bolts"? Or, has the significance of such an approach been unattractively garbed and therefore unfairly handicapped? Can we bridge the gap, rather than sever the link, between traditional ratio "analysis" and the more rigorous statistical techniques which have become popular among academicians in recent years?

The purpose of this paper is to attempt an assessment of this issue—the quality of ratio analysis as an analytical technique. The prediction of corporate bankruptcy is used as an illustrative case.¹ Specifically, a set of financial and economic ratios will be investigated in a bankruptcy prediction context wherein a multiple discriminant statistical methodology is employed. The data used in the study are limited to manufacturing corporations.

A brief review of the development of traditional ratio analysis as a technique for investigating corporate performance is presented in section I. In section II the shortcomings of this approach are discussed and multiple discriminant analysis is introduced with the emphasis centering on its compatibility with ratio analysis in a bankruptcy prediction context. The discriminant model is developed in section III, where an initial sample of sixty-six firms is utilized to establish a function which best discriminates between companies in two mutually exclusive groups: bankrupt and non-bankrupt firms. Section IV reviews empirical results obtained from the initial sample and several secondary samples, the latter being selected to examine the reliability of the discriminant

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1. In this study the term bankruptcy will, except where otherwise noted, refer to those firms that are legally bankrupt and either placed in receivership or have been granted the right to reorganize under the provisions of the National Bankruptcy Act.

model as a predictive technique. In section V the model's adaptability to practical decision-making situations and its potential benefits in a variety of situations are suggested. The final section summarizes the findings and conclusions of the study, and assesses the role and significance of traditional ratio analysis within a modern analytical context.

I. TRADITIONAL RATIO ANALYSIS

The detection of company operating and financial difficulties is a subject which has been particularly susceptible to financial ratio analysis. Prior to the development of quantitative measures of company performance, agencies were established to supply a qualitative type of information assessing the credit-worthiness of particular merchants.² Formal aggregate studies concerned with portents of business failure were evident in the 1930's. A study at that time³ and several later ones concluded that failing firms exhibit significantly different ratio measurements than continuing entities.⁴ In addition, another study was concerned with ratios of large asset-size corporations that experienced difficulties in meeting their fixed indebtedness obligations.⁵ A recent study involved the analysis of financial ratios in a bankruptcy-prediction context.⁶ This latter work compared a list of ratios individually for failed firms and a matched sample of non-failed firms. Observed evidence for five years prior to failure was cited as conclusive that ratio analysis can be useful in the prediction of failure.

The aforementioned studies imply a definite potential of ratios as predictors of bankruptcy. In general, ratios measuring profitability, liquidity, and solvency prevailed as the most significant indicators. The order of their importance is not clear since almost every study cited a different ratio as being the most effective indication of impending problems.

II. MULTIPLE DISCRIMINANT ANALYSIS

The previous section cited several studies devoted to the analysis of a firm's condition prior to financial difficulties. Although these works established certain important generalizations regarding the performance and trends of particular measurements, the adaptation of their results for assessing bankruptcy

2. For instance, the forerunner of well known Dun & Bradstreet, Inc. was organized in 1849 in Cincinnati, Ohio, in order to provide independent credit investigations. For an interesting and informative discussion on the development of credit agencies and financial measures of company performance see, Roy A. Foulke, *Practical Financial Statement Analysis*, 5th Ed., (New York, McGraw-Hill, 1961).

3. R. F. Smith and A. H. Winakor, *Changes in the Financial Structure of Unsuccessful Corporations*. (University of Illinois: Bureau of Business Research, 1935).

4. For instance, a comprehensive study covering over 900 firms compared discontinuing firms with continuing ones, see C. Merwin, *Financing Small Corporations* (New York: Bureau of Economic Research, 1942).

5. W. B. Hickman, *Corporate Bond Quality and Investor Experience* (Princeton, N.J.: Princeton University Press, 1958).

6. W. H. Beaver, "Financial Ratios as Predictors of Failure," *Empirical Research in Accounting, Selected Studies*, 1966 (Institute of Professional Accounting, January, 1967), pp. 71-111. Also a recent attempt was made to weight ratios arbitrarily, see M. Tamari, "Financial Ratios as a Means of Forecasting Bankruptcy," *Management International Review*, Vol. 4 (1966), pp. 15-21.

potential of firms, both theoretically and practically, is questionable.⁷ In almost every case, the methodology was essentially univariate in nature and emphasis was placed on individual signals of impending problems.⁸ Ratio analysis presented in this fashion is susceptible to faulty interpretation and is potentially confusing. For instance, a firm with a poor profitability and/or solvency record may be regarded as a potential bankrupt. However, because of its above average liquidity, the situation may not be considered serious. The potential ambiguity as to the relative performance of several firms is clearly evident. The crux of the shortcomings inherent in any univariate analysis lies therein. An appropriate extension of the previously cited studies, therefore, is to build upon their findings and to combine several measures into a meaningful predictive model. In so doing, the highlights of ratio analysis as an analytical technique will be emphasized rather than downgraded. The question becomes, which ratios are most important in detecting bankruptcy potential, what weights should be attached to those selected ratios, and how should the weights be objectively established.

After careful consideration of the nature of the problem and of the purpose of the paper, a multiple discriminant analysis (MDA) was chosen as the appropriate statistical technique. Although not as popular as regression analysis, MDA has been utilized in a variety of disciplines since its first application in the 1930's.⁹ During those earlier years MDA was used mainly in the biological and behavioral sciences.¹⁰ More recently this method had been applied successfully to financial problems such as consumer credit evaluation¹¹ and investment classification. For instance in the latter area, Walter utilized a MDA model to classify high and low price earnings ratio firms,¹² and Smith applied the technique in the classification of firms into standard investment categories.¹³

MDA is a statistical technique used to classify an observation into one of several *a priori* groupings dependent upon the observation's individual characteristics. It is used primarily to classify and/or make predictions in problems

7. At this point bankruptcy is used in its most general sense, meaning simply business failure.

8. Exceptions to this generalization were noted in works where there was an attempt to emphasize the importance of a group of ratios as an indication of overall performance. For instance, Foulke, *op. cit.*, chapters XIV and XV, and A. Wall and R. W. Duning, *Ratio Analysis of Financial Statements*, (New York: Harper and Row, 1928), p. 159.

9. R. A. Fisher, "The Use of Multiple Measurements in Taxonomic Problems," *Annals of Eugenics*, No. 7 (September, 1936), pp. 179-188.

10. For a comprehensive review of studies using MDA see W. G. Cochran, "On the Performance of the Linear Discriminant Function," *Technometrics*, vol. 6 (May, 1964), pp. 179-190.

11. The pioneering work utilizing MDA in a financial context was performed by Durand in evaluating the credit worthiness of used car loan applicants, see D. D. Durand, *Risk Elements in Consumer Installment Financing*, Studies in Consumer Installment Financing (New York: National Bureau of Economic Research, 1941), pp. 105-142. More recently, Myers and Forgy analyzed several techniques, including MDA, in the evaluation of good and bad installment loans, see H. Myers and E. W. Forgy, "Development of Numerical Credit Evaluation Systems," *Journal of American Statistical Association*, vol. 50 (September, 1963), pp. 797-806.

12. J. E. Walter, "A Discriminant Function for Earnings Price Ratios of Large Industrial Corporations," *Review of Economics and Statistics*, vol. XLI (February, 1959), pp. 44-52.

13. K. V. Smith, *Classification of Investment Securities Using MDA*, Institute Paper #101 (Purdue University, Institute for Research in the Behavioral, Economic, and Management Sciences, 1965).

where the dependent variable appears in qualitative form, e.g., male or female, bankrupt or non-bankrupt. Therefore, the first step is to establish explicit group classifications. The number of original groups can be two or more.

After the groups are established, data are collected for the objects in the groups; MDA then attempts to derive a linear combination of these characteristics which "best" discriminates between the groups. If a particular object, for instance a corporation, has characteristics (financial ratios) which can be quantified for all of the companies in the analysis, the MDA determines a set of discriminant coefficients. When these coefficients are applied to the actual ratio, a basis for classification into one of the mutually exclusive groupings exists. The MDA technique has the advantage of considering an entire profile of characteristics common to the relevant firms, as well as the interaction of these properties. A univariate study, on the other hand, can only consider the measurements used for group assignments one at a time.

Another advantage of MDA is the reduction of the analyst's space dimensionality, i.e., from the number of different independent variables to $G - 1$ dimension(s), where G equals the number of original *a priori* groups.¹⁴ This paper is concerned with two groups, consisting of bankrupt firms on the one hand, and of non-bankrupt firms on the other. Therefore, the analysis is transformed into its simplest form: one dimension. The discriminant function of the form $Z = v_1 x_1 + v_2 x_2 + \dots + v_n x_n$ transforms individual variable values to a single discriminant score or Z value which is then used to classify the object

where $v_1, v_2, \dots, v_n =$ Discriminant coefficients

$x_1, x_2, \dots, x_n =$ Independent variables

The MDA computes the discriminant coefficients, v_j , while the independent variables x_j are the actual values

where, $j = 1, 2, \dots, n$.

When utilizing a comprehensive list of financial ratios in assessing a firm's bankruptcy potential there is reason to believe that some of the measurements will have a high degree of correlation or collinearity with each other. While this aspect necessitates careful selection of the predictive variables (ratios), it also has the advantage of yielding a model with a relatively small number of selected measurements which has the potential of conveying a great deal of information. This information might very well indicate differences between groups but whether or not these differences are significant and meaningful is a more important aspect of the analysis. To be sure, there are differences between bankrupt firms and healthy ones; but are these differences of a magnitude to facilitate the development of an accurate prediction model?

Perhaps the primary advantage of MDA in dealing with classification problems is the potential of analyzing the entire variable profile of the object simultaneously rather than sequentially examining its individual characteristics.

14. For a formulation of the mathematical computations involved in MDA, see J. G. Bryan, "The Generalized Discriminant Function, Mathematical Foundation & Computational Routine," *Harvard Educational Review*, vol. XXI, no. 2 (Spring, 1951), pp. 90-95, and C. R. Rao, *Advanced Statistical Methods in Biometric Research* (New York: John Wiley & Sons, Inc., 1952).

Just as linear and integer programming have improved upon traditional techniques in capital budgeting¹⁵ the MDA approach to traditional ratio analysis has the potential to reformulate the problem correctly. Specifically, combinations of ratios can be analyzed together in order to remove possible ambiguities and misclassifications observed in earlier traditional studies.

Given the above descriptive qualities, the MDA technique was selected as most appropriate for the bankruptcy study. A carefully devised and interpreted multiple regression analysis methodology conceivably could have been used in this two group case.

III. DEVELOPMENT OF THE MODEL

Sample Selection. The initial sample is composed of sixty-six corporations with thirty-three firms in each of the two groups. The bankrupt group (1) are manufacturers that filed a bankruptcy petition under Chapter X of the National Bankruptcy Act during the period 1946-1965.¹⁶ The mean asset size of these firms is \$6.4 million, with a range of between \$0.7 million and \$25.9 million. Recognizing that this group is not completely homogeneous, due to industry and size differences, a careful selection of non-bankrupt firms was attempted. Group 2 consisted of a paired sample of manufacturing firms chosen on a stratified random basis. The firms are stratified by industry and by size, with the asset size range restricted to between \$1-\$25 million.¹⁷ Firms in Group 2 were still in existence in 1966. Also, the data collected are from the same years as those compiled for the bankrupt firms. For the initial sample test, the data are derived from financial statements one reporting period prior to bankruptcy.¹⁸

An important issue is to determine the asset-size group to be sampled. The decision to eliminate both the small firms (under \$1 million in total assets) and the very large companies from the initial sample essentially is due to the asset range of the firms in Group 1. In addition, the incidence of bankruptcy in the large asset-size firm is quite rare today while the absence of comprehensive data negated the representation of small firms. A frequent argument is that financial ratios, by their very nature, have the effect of deflating statistics by size, and therefore a good deal of the size effect is eliminated. To choose Group 1 firms in a restricted size range is not feasible, while selecting firms for Group 2 at random seemed unwise. However, subsequent tests to the original sample do not use size as a means of stratification.¹⁹

15. H. M. Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting, Budgeting Problems*, (Englewood Cliffs, New Jersey: Prentice-Hall, 1963).

16. The choice of a twenty year period is not the best procedure since average ratios do shift over time. Ideally we would prefer to examine a list of ratios in time period t in order to make predictions about other firms in the following period ($t + 1$). Unfortunately it was not possible to do this because of data limitations. However, the number of bankruptcies were approximately evenly distributed over the twenty year period in both the original and the secondary samples.

17. The mean asset size of the firms in Group 2 (\$9.6 million) was slightly greater than that of Group 1, but matching exact asset size of the two groups seemed unnecessary.

18. The data was derived from Moody's Industrial Manuals and selected Annual Reports. The average lead time of the financial statements was approximately seven and one-half months prior to bankruptcy.

19. One of these tests included only firms that experienced operating losses (secondary sample of non-bankrupt firms).

After the initial groups are defined and firms selected, balance sheet and income statement data are collected. Because of the large number of variables found to be significant indicators of corporate problems in past studies, a list of twenty-two potentially helpful variables (ratios) is compiled for evaluation. The variables are classified into five standard ratio categories, including liquidity, profitability, leverage, solvency, and activity ratios. The ratios are chosen on the basis of their 1) popularity in the literature,²⁰ 2) potential relevancy to the study, and a few "new" ratios initiated in this paper.

From the original list of variables, five variables are selected as doing the best overall job together in the prediction of corporate bankruptcy.²¹ In order to arrive at a final profile of variables the following procedures are utilized: (1) Observation of the statistical significance of various alternative functions including determination of the relative contributions of each independent variable; (2) evaluation of inter-correlations between the relevant variables; (3) observation of the predictive accuracy of the various profiles; and (4) judgment of the analyst.

The variable profile finally established did not contain the most significant variables, amongst the twenty-two original ones, measured independently. This would not necessarily improve upon the univariate, traditional analysis described earlier. The contribution of the entire profile is evaluated, and since this process is essentially iterative, there is no claim regarding the optimality of the resulting discriminant function. The function, however, does the best job among the alternatives which include numerous computer runs analyzing different ratio-profiles. The final discriminant function is as follows:

$$(I) \quad Z = .012X_1 + .014X_2 + .033X_3 + .006X_4 + .999X_5$$

where X_1 = Working capital/Total assets

X_2 = Retained Earnings/Total assets

X_3 = Earnings before interest and taxes/Total assets

X_4 = Market value equity/Book value of total debt

X_5 = Sales/Total assets

Z = Overall Index

X_1 —*Working Capital/Total Assets*. The Working capital/Total assets ratio, frequently found in studies of corporate problems, is a measure of the net liquid assets of the firm relative to the total capitalization. Working capital is defined as the difference between current assets and current liabilities. Liquidity and size characteristics are explicitly considered. Ordinarily, a firm experiencing consistent operating losses will have shrinking current assets in relation to total assets. Of the three liquidity ratios evaluated, this one proved to be the most valuable.²² Inclusion of this variable is consistent with the Merwin study which

20. The Beaver study (cited earlier) concluded that the cash flow to debt ratio was the best single ratio predictor. This ratio was not considered here because of the lack of consistent appearance of precise depreciation data. The results obtained, however (see section IV), are superior to the results Beaver attained with his single best ratio, see Beaver, *op. cit.*, p. 89.

21. The MDA computer program used in this study was developed by W. Cooley and P. Lohnes. The data are organized in a blocked format; the bankrupt firms' data first followed by the non-bankrupt firms'.

22. The other two liquidity ratios were the current ratio and the quick ratio. The Working capital/Total assets ratio showed greater statistical significance both on a univariate and multivariate basis.

rated the net working capital to total asset ratio as the best indicator of ultimate discontinuance.²³

X_2 —*Retained Earnings/Total Assets*.²⁴ This measure of cumulative profitability over time was cited earlier as one of the "new" ratios. The age of a firm is implicitly considered in this ratio. For example, a relatively young firm will probably show a low RE/TA ratio because it has not had time to build up its cumulative profits. Therefore, it may be argued that the young firm is somewhat discriminated against in this analysis, and its chance of being classified as bankrupt is relatively higher than another, older firm, *ceteris paribus*. But, this is precisely the situation in the real world. The incidence of failure is much higher in a firm's earlier years.²⁵

X_3 —*Earnings Before Interest and Taxes/Total Assets*. This ratio is calculated by dividing the total assets of a firm into its earnings before interest and tax reductions. In essence, it is a measure of the true productivity of the firm's assets, abstracting from any tax or leverage factors. Since a firm's ultimate existence is based on the earning power of its assets, this ratio appears to be particularly appropriate for studies dealing with corporate failure. Furthermore, insolvency in a bankruptcy sense occurs when the total liabilities exceed a fair valuation of the firm's assets with value determined by the earning power of the assets.

X_4 —*Market Value of Equity/Book Value of Total Debt*. Equity is measured by the combined market value of all shares of stock, preferred and common, while debt includes both current and long-term. The measure shows how much the firm's assets can decline in value (measured by market value of equity plus debt) before the liabilities exceed the assets and the firm becomes insolvent. For example, a company with a market value of its equity of \$1,000 and debt of \$500 could experience a two-thirds drop in asset value before insolvency. However, the same firm with \$250 in equity will be insolvent if its drop is only one-third in value. This ratio adds a market value dimension which other failure studies did not consider.²⁶ It also appears to be a more effective predictor of bankruptcy than a similar, more commonly used ratio: Net worth/Total debt (book values).

X_5 —*Sales/Total Assets*. The capital-turnover ratio is a standard financial ratio illustrating the sales generating ability of the firm's assets. It is one measure of management's capability in dealing with competitive conditions. This final ratio is quite important because, as indicated below, it is the least

23. Merwin, *op. cit.*, p. 99.

24. Retained Earnings is the account which reports the total amount of reinvested earnings and/or losses of a firm over its entire life. The account is also referred to as Earned Surplus. It should be noted that the Retained Earnings account is subject to manipulation via corporate quasi-reorganizations and stock dividend declarations. While these occurrences are not evident in this study it is conceivable that a bias would be created by a substantial reorganization or stock dividend.

25. In 1965, over 50 per cent of all manufacturing firms that failed did so in the first five years of their existence. Over 31 per cent failed within three years. Statistics taken from *The Failure Record, Through 1965* (New York: Dun & Bradstreet, Inc., 1966), p. 10.

26. The reciprocal of X_4 is the familiar Debt/Equity ratio often used as a measure of financial leverage. X_4 is a slightly modified version of one of the variables used effectively by Fisher in a study of corporate bond interest rate differentials, see Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," *Journal of Political Economy*, LXVII, No. 3 (June, 1959), pp. 217-237.

significant ratio on an individual basis. In fact, based on the statistical significance measure, it would not have appeared at all. However, because of its unique relationship to other variables in the model, the Sales/Total assets ratio ranks second in its contribution to the overall discriminating ability of the model.

To test the individual discriminating ability of the variables, an "F" test is performed. This test relates the difference between the average values of the ratios in each group to the variability (or spread) of values of the ratios within each group. Variable means one financial statement prior to bankruptcy and the resulting "F" statistics are presented in Table 1.

TABLE 1
VARIABLE MEANS AND TEST OF SIGNIFICANCE

Variable	Bankrupt Group Mean	Non-Bankrupt Group Mean	F Ratio
	n = 33	n = 33	
X ₁	- 6.1%	41.4%	32.60*
X ₂	-62.6%	35.5%	58.86*
X ₃	-31.8%	15.3%	26.56*
X ₄	40.1%	247.7%	33.26*
X ₅	150.0%	190.0%	2.84

* Significant at the .001 level.

F_{1,60} (.001) = 12.00

F_{1,60} (.01) = 7.00

F_{1,60} (.05) = 4.00

Variables X₁ through X₄ are all significant at the .001 level, indicating extremely significant differences in these variables between groups. Variable X₅ does not show a significant difference between groups and the reason for its inclusion in the variable profile is not apparent as yet. On a strictly univariate level, all of the ratios indicate higher values for the non-bankrupt firms. Also, the discriminant coefficients of equation (I) display positive signs, which is what one would expect. Therefore, the greater a firm's bankruptcy potential, the lower its discriminant score.

One useful technique in arriving at the final variable profile is to determine the relative contribution of each variable to the total discriminating power of the function, and the interaction between them. The relevant statistic is observed as a scaled vector which is computed by multiplying corresponding elements by the square roots of the diagonal elements of the variance-co-variance matrix.²⁷ Since the actual variable measurement units are not all comparable to each other, simple observation of the discriminant coefficients is misleading. The adjusted coefficients shown in Table 2 enable us to evaluate each variable's contribution on a relative basis.

The scaled vectors indicate that the large contributors to group separation

27. For example, the square root of the appropriate variance-covariance figure (standard deviation) for X₁ is approximately .275 and when multiplied by the variable's coefficient (.012) yields a scaled vector of 3.29.

of the discriminant function are X_3 , X_5 , and X_4 , respectively. The profitability ratio contributes the most, which is not surprising if one considers that the incidence of bankruptcy in a firm that is earning a profit is almost nil. What is surprising, however, is the second highest contribution of X_5 (Sales/Total assets). Recalling that this ratio was insignificant on a univariate basis, the multivariate context is responsible for illuminating the importance of X_5 .²⁸ A probable reason for this unexpected result is the high negative correlation (-0.78) we observe between X_3 and X_5 in the bankruptcy group. The negative correlation is also evident in subsequent bankrupt group samples.

TABLE 2
RELATIVE CONTRIBUTION OF THE VARIABLES

Variable	Scaled Vector	Ranking
X_1	3.29	5
X_2	6.04	4
X_3	9.89	1
X_4	7.42	3
X_5	8.41	2

In a recent evaluation of the discriminant function, Cochran concluded that most correlations between variables in past studies were positive and that, by and large, negative correlations are more helpful than positive correlations in adding new information to the function.²⁹ The logic behind the high negative correlation in the bankrupt group is that as firms suffer losses and deteriorate toward failure, their assets are not replaced as much as in healthier times, and also the cumulative losses have further reduced the asset size through debits to Retained Earnings. The asset size reduction apparently dominates any sales movements.

A different argument, but one not necessarily inconsistent with the above, concerns a similar ratio to X_5 , Net Sales to Tangible Net Worth. If the latter ratio is excessive the firm is often referred to as a poor credit risk due to insufficient capital to support sales. Companies with moderate or even below average sales generating lower (low asset turnover, X_5) might very well possess an extremely high Net Sales/Net Worth ratio if the Net Worth has been reduced substantially due to cumulative operating losses. This ratio, and other net worth ratios, are not considered in the paper because of computational and interpretive difficulties arising when negative net worth totals are present.

It is clear that four of the five variables display significant differences between groups, but the importance of MDA is its ability to separate groups using multivariate measures. A test to determine the overall discriminating power of the model is the common F-value which is the ratio of the sums-of-squares

28. For an excellent discussion of how a seemingly insignificant variable on a univariate basis can supply important information in a multivariate context see, W. W. Cooley and P. R. Lohnes *Multivariate Procedures for the Behavioral Sciences* (New York: John Wiley and Sons, Inc., 1962), p. 121.

29. Cochran, *op. cit.*, p. 182.

between-groups to the within-groups sums-of-squares. When this ratio of the form,

$$\lambda = \frac{\sum_{g=1}^G N_g [\bar{y}_g - \bar{y}]^2}{\sum_{g=1}^G \sum_{p=1}^{N_g} [y_{pg} - \bar{y}_g]^2}$$

where

- G = Number of groups
- g = Group g, g = 1 . . . G
- N_g = Number of firms in group g
- y_{pg} = Firm p in group g, p = 1 . . . N_g
- \bar{y}_g = Group mean (centroid)
- \bar{y} = Overall sample mean

is maximized, it has the effect of spreading the means (centroids) of the G groups apart and, simultaneously, reducing dispersion of the individual points (firm Z values, y_{pg}) about their respective group means. Logically, this test (commonly called the "F" test) is appropriate because one of the objectives of the MDA is to identify and to utilize those variables which best discriminate *between* groups and which are most similar *within* groups.

The group means, or centroids, of the original two-group sample of the form

$$\bar{y}_g = \frac{1}{N_g} \sum_{p=1}^{N_g} y_{pg}$$

are

Group 1 = -0.29	F = 20.7
Group 2 = +5.02	F _{5,60} (.01) = 3.34

The significance test therefore *rejects* the null hypothesis that the observations come from the same population. With the conclusion that *a priori* groups are significantly different, further discriminatory analysis is possible.

Once the values of the discriminant coefficients are estimated, it is possible to calculate discriminant scores for each observation in the sample, or any firm, and to assign the observations to one of the groups based on this score. The essence of the procedure is to compare the profile of an individual firm with that of the alternative groupings. In this manner the firm is assigned to the group it most closely resembles. The comparisons are measured by a chi-square value and assignments are made based upon the relative proximity of the firm's score to the various group centroids.

IV. EMPIRICAL RESULTS

At the outset, it might be helpful to illustrate the format for presenting the results. In the multi-group case, results are shown in a classification chart or "accuracy-matrix." The chart is set up as follows:

Actual Group Membership	Predicted Group Membership	
	Bankrupt	Non-Bankrupt
Bankrupt	H	M ₁
Non-Bankrupt	M ₂	H

The actual group membership is equivalent to the *a priori* groupings and the model attempts to classify correctly these firms. At this stage, the model is basically explanatory. When new companies are classified, the nature of the model is predictive.

The H's stand for correct classifications (Hits) and the M's stand for misclassifications (Misses). M₁ represents a Type I error and M₂ a Type II error. The sum of the diagonal elements equals the total correct "hits," and when divided into the total number of firms classified (sixty-six in the case of the initial sample), yields the measure of success of the MDA in classifying firms, that is, the per cent of firms correctly classified. This percentage is analogous to the coefficient of determination (R²) in regression analysis, which measures the per cent of the variation of the dependent variable explained by the independent variables.

The final criterion used to establish the best model was to observe its accuracy in predicting bankruptcy. A series of six tests were performed.

(1) *Initial Sample (Group 1)*. The initial sample of 33 firms in each of the two groups is examined using data one financial statement prior to bankruptcy. Since the discriminant coefficients and the group distributions are derived from this sample, a high degree of successful classification is expected. This should occur because the firms are classified using a discriminant function which, in fact, is based upon the individual measurements of these same firms. The classification matrix for the initial sample is as follows:

	Actual	Predicted	
		Group 1	Group 2
Group 1		31	2
Group 2		1	32

	Number Correct	Per cent Correct	Per cent Error	n
Type I	31	94	6	33
Type II	32	97	3	33
Total	63	95	5	66

The model is extremely accurate in classifying 95 per cent of the total sample correctly. The *Type I error* proved to be only 6 per cent, while the *Type II error* was even better at 3 per cent. The results, therefore, are encouraging, but the obvious upward bias should be kept in mind and further validation techniques are appropriate.

(2) *Results Two Years Prior to Bankruptcy*. The second test is made to observe the discriminating ability of the model for firms using data from two

years prior to bankruptcy. The two year period is an exaggeration since the average lead time for the correctly classified firms is approximately twenty months with two firms having a thirteen month lead. The results are:

	Number Correct	Per cent Correct	Per cent Error	n	Predicted	
					Group 1 (Bankrupt)	Group 2 (Non-Bankrupt)
Group 1					23	9
Group 2					2	31
Type I	23	72	28	32		
Type II	31	94	6	33		
Total	54	83	17	65		

The reduction in the accuracy of group classification is understandable because impending bankruptcy is more remote and the indications are less clear. Nevertheless, 72 per cent correct assignment is evidence that bankruptcy can be predicted two years prior to the event. The Type II error is slightly larger (6 per cent vs. 3 per cent) in this test but still is extremely accurate. Further tests will be applied below to determine the accuracy of predicting bankruptcy as much as five years prior to the actual event.

(3) *Potential Bias and Validation Techniques.* When the firms used to determine the discriminant coefficients are re-classified, the resulting accuracy is biased upward by (a) sampling errors in the original sample and (b) search bias. The latter bias is inherent in the process of reducing the original set of variables (twenty-two) to the best variable profile (five). The possibility of bias due to intensive searching is inherent in any empirical study. While a subset of variables is effective in the initial sample, there is no guarantee that it will be effective for the population in general.

The importance of secondary sample testing cannot be over-emphasized and it appears appropriate to apply these measures at this stage. A method suggested by Frank *et al.*³⁰ for testing the extent of the aforementioned search bias was applied to the initial sample. The essence of this test is to estimate parameters for the model using only a subset of the original sample, and then to classify the remainder of the sample based on the parameters established. A simple t-test is then applied to test the significance of the results.

Five different replications of the suggested method of choosing subsets (sixteen firms) of the original sample are tested, with results listed in Table 3.³¹

The test results reject the hypothesis that there is no difference between the groups and substantiate that the model does, in fact, possess discriminating

30. R. E. Frank, W. F. Massy, and G. D. Morrison, "Bias in Multiple Discriminant Analysis," *Journal of Marketing Research*, vol. 2 (August 1965), pp. 250-258.

31. The five replications included (1) random sampling (2) choosing every other firm starting with firm number one, (3) starting with firm number two, (4) choosing firms 1-16, and (5) firms 17-32.

power on observations other than those used to establish the parameters of the model. Therefore, any search bias does not appear significant.

TABLE 3
ACCURACY OF CLASSIFYING A SECONDARY SAMPLE

Replication	Per cent of Correct Classifications	Value of t
1	91.2	4.8*
2	91.2	4.8*
3	97.0	5.5*
4	97.0	4.5*
5	91.2	4.8*
Average	93.5	5.1*
Total number of observations per replication		34

* Significant at the .001 level.

$$t = \frac{\text{proportion correct} - .5}{\sqrt{\frac{.5(1 - .5)}{n}}}$$

(4) *Secondary Sample of Bankrupt Firms.* In order to test the model rigorously for both bankrupt and non-bankrupt firms two new samples are introduced. The first contains a new sample of twenty-five bankrupt firms whose asset-size range is the same as that of the initial bankrupt group. Using the parameters established in the discriminant model to classify firms in this secondary sample, the predictive accuracy for this sample as of one statement prior to bankruptcy is:

Bankrupt Group (Actual)	Number Correct	Predicted		n
		Bankrupt	Non-Bankrupt	
		24	1	
Type I (total)	24	96	4	25

The results here are surprising in that one would not usually expect a secondary sample's results to be superior to the initial discriminant sample (96 per cent vs. 94 per cent). Two possible reasons are that the upward bias normally present in the initial sample tests is not manifested in this investigation, and/or the model, as stated before, is something less than optimal.

(5) *Secondary Sample of Non-Bankrupt Firms.* Up to this point the sample companies were chosen either by their bankruptcy status (Group 1) or by their similarity to Group 1 in all aspects except their economic well-being. But what of the many firms which suffer temporary profitability difficulties, but in actuality do not become bankrupt. A bankruptcy classification of a firm from this group is an example of a *Type II error*. An exceptionally rigorous

test of the discriminant model's effectiveness would be to search out a large sample of firms that have encountered earnings problems and then to observe the MDA's classification results.

In order to perform the above test, a sample of sixty-six firms is selected on the basis of net income (deficit) reports in the years 1958 and 1961, with thirty-three from each year. Over 65 per cent of these firms had suffered two or three years of negative profits in the previous three years reporting. The firms are selected regardless of their asset size, with the only two criteria being that they were manufacturing firms which suffered losses in the year 1958 or 1961.³² The two base years are chosen due to their relatively poor economic performances in terms of GNP growth. The companies are then evaluated by the discriminant model to determine their predictive bankruptcy potential.

The results, illustrated below, show that fifteen of the sixty-six firms are classified as bankrupts with the remaining fifty-one correctly classified. The number of misclassifications is actually fourteen, as one of the firms went bankrupt within two years after the data period.

Non-Bankrupt Group Actual	Number Correct	Predicted		n
		Bankrupt	Non-Bankrupt	
		14	52	
		Per cent Correct	Per cent Error	
Type II (total)	52	79	21	66

Therefore, the discriminant model correctly classified 79 per cent of the sample firms. This percentage is all the more impressive when one considers that these firms constitute a *secondary* sample of admittedly *below average* performance. The t-test for the significance of this result is $t = 4.8$; significant at the .001 level.

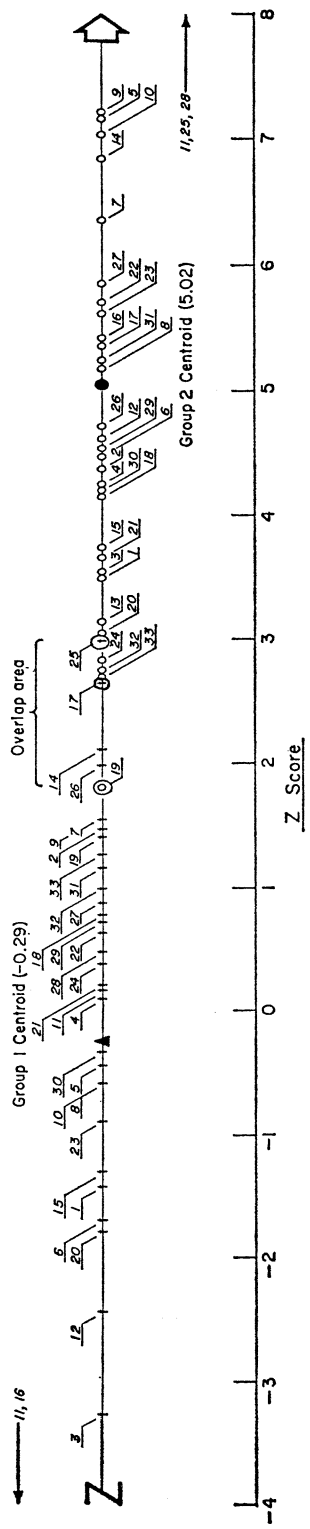
Another interesting facet of this test is the relationship of these "temporarily" sick firms' Z scores, and the "zone of ignorance" or gray-area described more completely in the next section. Briefly, the "zone of ignorance" is that range of Z scores (see Chart I) where misclassifications can be observed. Chart I illustrates some of the individual firm Z scores (initial sample) and the group centroids. These points are plotted in one dimensional space and, therefore, are easily visualized.

Of the fourteen misclassified firms in this secondary sample, ten have Z scores between 1.81 and 2.67, which indicates that although they are classified as bankrupts, the prediction of bankruptcy is not as definite as the vast majority in the initial sample of bankrupt firms. In fact, just under one-third of the sixty-six firms in this last sample have Z scores within the entire overlap area, which emphasizes that the selection process is successful in choosing firms which showed signs (profitability) of deterioration.

32. The firms were selected at random from all the firms listed in *Standard and Poor's Stock Guide*, January 1959, 1962, that reported negative earnings.

Chart I INDIVIDUAL FIRM DISCRIMINANT SCORES AND GROUP CENTROIDS---ONE YEAR PRIOR TO BANKRUPTCY

$$(Z = .012 X_1 + .014 X_2 + .033 X_3 + .006 X_4 + .999 X_5)$$



KEY:

- ▲ = Discriminate Points (Group 1 - Bankrupt Firms) n = 33
 - = Discriminate Points (Group 2 - Non-bankrupt Firms) n = 33
 - ⊙ = Misclassified Firms (Group 1) = 2
 - ⊙ = Misclassified Firms (Group 2) = 1
- } one year prior

(6) *Long-Range Predictive Accuracy.* The previous results give important evidence of the reliability of the conclusions derived from the initial sample of firms. An appropriate extension, therefore, would be to examine the firms to determine the overall effectiveness of the discriminant model for a longer period of time prior to bankruptcy. Several studies, e.g., Beaver and Merwin, indicated that their analyses showed firms exhibiting failure tendencies as much as five years prior to the actual failure. Little is mentioned, however, of the true significance of these earlier year results. Is it enough to show that a firm's position is deteriorating or is it more important to examine when in the life of a firm does its eventual failure, if any, become an acute possibility? Thus far, we have seen that bankruptcy can be predicted accurately for two years prior to failure. What about the more remote years?

To answer this question, data are gathered for the thirty-three original firms from the third, fourth, and fifth year prior to bankruptcy. The reduced sample is due to the fact that several of the firms were in existence for less than five years. In two cases data were unavailable for the more remote years. One would expect on an *a priori* basis that, as the lead time increases, the relative predictive ability of any model would decrease. This was true in the univariate studies cited earlier, and it is also quite true for the multiple discriminant model. Table 4 summarizes the predictive accuracy for the total five year period.

TABLE 4
FIVE YEAR PREDICTIVE ACCURACY OF THE MDA MODEL
(Initial Sample)

Year Prior to Bankruptcy	Hits	Misses	Per cent Correct
1st n = 33	31	2	95
2nd n = 32	23	9	72
3rd n = 29	14	15	48
4th n = 28	8	20	29
5th n = 25	9	16	36

It is obvious that the accuracy of the model falls off consistently with the one exception of the fourth and fifth years, when the results are reversed from what would be expected. The most logical reason for this occurrence is that after the second year, the discriminant model becomes unreliable in its predictive ability, and, also, that the change from year to year has little or no meaning.

Implications. Based on the above results it is suggested that the bankruptcy prediction model is an accurate forecaster of failure up to two years prior to bankruptcy and that the accuracy diminishes substantially as the lead time increases. In order to investigate the possible reasons underlying these findings the trend in the five predictive variables is traced on a univariate basis for five years preceding bankruptcy. The ratios of four other important but less significant ratios are also listed in Table 5.

The two most important conclusions of this trend analysis are (1) that all of the observed ratios show a deteriorating trend as bankruptcy approached,

TABLE 5
AVERAGE RATIOS OF BANKRUPT GROUP PRIOR TO FAILURE—ORIGINAL SAMPLE

Ratio	Fifth Year		Fourth Year		Third Year		Second Year		First Year	
	Ratio	Change ^a	Ratio	Change ^a	Ratio	Change ^a	Ratio	Change ^a	Ratio	Change ^a
Working Capital/Total Assets (%) (X ₁)	19.5	+ 3.6	23.2	+ 3.6	17.6	- 5.6	1.6	-16.0 ^b	(6.1)	- 7.7
Retained Earnings/Total Assets (%) (X ₂)	4.0	- 4.8	(0.8)	- 4.8	(7.0)	- 6.2	(30.1)	-23.1	(62.6)	-32.5 ^b
EBIT/Total Assets (%) (X ₃)	7.2	- 3.2	4.0	- 3.2	(5.8)	- 9.8	(20.7)	-14.9 ^b	(31.8)	-11.1
Market Value Equity/Total Debt (%) (X ₄)	180.0	-32.4	147.6	-32.4	143.2	- 4.4	74.2	-69.0 ^b	40.1	-34.1
Sales/Total Assets (%) (X ₅)	200.0	0.0	200.0	0.0	166.0	-34.0 ^b	150.0	-16.0	150.0	0.0
Current Ratio (%)	180.0	+ 7.0	187.0	+ 7.0	162.0	-25.0	131.0	-31.0 ^b	133.0	+ 2.0
Years of Negative Profits (yrs.)	0.8	+ 0.1	0.9	+ 0.1	1.2	+ 0.3	2.0	+ 0.8 ^b	2.5	+ 0.5
Total Debt/Total Assets (%)	54.2	+ 6.7	60.9	+ 6.7	61.2	+ 0.3	77.0	+15.8	96.4	+19.4 ^b
Net Worth/Total Debt (%)	123.2	-28.0	75.2	-28.0	112.6	+17.4	70.5	-42.1 ^b	49.4	-21.1

^a Change from previous year.

^b Largest yearly change in the ratio.

and (2) that the most serious change in the majority of these ratios occurred between the third and the second years prior to bankruptcy. The degree of seriousness is measured by the yearly change in the ratio values. The latter observation is extremely significant as it provides evidence consistent with conclusions derived from the discriminant model. Therefore, the important information inherent in the individual ratio measurement trends takes on deserved significance only when integrated with the more analytical discriminant analysis findings.

V. APPLICATIONS

The use of a multiple discriminant model for predicting bankruptcy has displayed several advantages, but bankers, credit managers, executives, and investors will typically not have access to computer procedures such as the Cooley-Lohnes MDA program. Therefore, it will be necessary to investigate the results presented in Section IV closely and to attempt to extend the model for more general application. The procedure described below may be utilized to select a "cut-off" point, or optimum Z value, which enables predictions without computer support.³³

By observing those firms which have been misclassified by the discriminant model in the initial sample, it is concluded that all firms having a Z score of greater than 2.99 clearly fall into the "non-bankrupt" sector, while those firms having a Z below 1.81 are all bankrupt. The area between 1.81 and 2.99 will be defined as the "zone of ignorance" or "gray area" because of the susceptibility to error classification (see Chart I). Since errors are observed in this range of values, we will be uncertain about a *new* firm whose Z value falls within the "zone of ignorance." Hence, it is desirable to establish a guideline for classifying firms in the "gray area."

The process begins by identifying sample observations which fall within the overlapping range. These appear as in Table 6. The first digit of the firm

TABLE 6
FIRM WHOSE Z SCORE FALLS WITHIN GRAY AREA

Firm Number Non-Bankrupt	Z Score	Firm Number Bankrupt
2019*	1.81	
	1.98	1026
	2.10	1014
	2.67	1017*
	2.68	
2033	2.78	
2032	2.99	1025*

* Misclassified by the MDA model; for example, firm "19" in Group 2.

number identifies the group, with the last two digits locating the firm within the group.

33. A similar method proved to be useful in selecting cut-off points for marketing decisions, see R. E. Frank, A. A. Kuehn, W. F. Massy, *Quantitative Techniques in Marketing Analysis* (Homewood, Ill.: Richard D. Irwin, Inc., 1962), pp. 95-100.

Next, the range of values of Z that results in the *minimum number of misclassifications* is found. In the analysis, Z's between (but not including) the indicated values produce the following misclassifications as shown in Table 7.

TABLE 7
NUMBER OF MISCLASSIFICATIONS USING VARIOUS Z SCORE CRITERIONS

Range of Z	Number Misclassified	Firms
1.81-1.98	5	2019, 1026, 1014, 1017, 1025
1.98-2.10	4	2019, 1014, 1017, 1025
2.10-2.67	3	2019, 1017, 1025
2.67-2.68	2	2019, 1025
2.68-2.78	3	2019, 2033, 1025
2.78-2.99	4	2019, 2033, 2032, 1025

The best critical value conveniently falls between 2.67-2.68 and therefore 2.675, the midpoint of the interval, is chosen as the Z value that discriminates best between the bankrupt and non-bankrupt firms.

Of course, the real test of this "optimum" Z value is its discriminating power not only with the initial sample, but also with the secondary samples. The results of these tests are even slightly superior to the job done by the computer assignments, with the additional benefit of practical applicability.

Business Loan Evaluation. Reference was made earlier to several studies which examined the effectiveness of discriminant analysis in evaluating *consumer-loan* applications and, perhaps, these suggest a useful extension of the bankruptcy-prediction model. The evaluation of *business-loans* is an important function in our society, especially to commercial banks and other lending institutions. Studies have been devoted to the loan offer function³⁴ and to the adoption of a heuristic-bank-loan-officer model whereby a computer model was developed to simulate the loan officer function.³⁵ Admittedly, the analysis of the loan applicant's financial statements is but one section of the entire evaluation process, but it is a very important link. A fast and efficient device for detecting unfavorable credit risks might enable the loan officer to avoid potentially disastrous decisions. The significant point is that the MDA model contains many of the variables common to business-loan evaluation and discriminant analysis has been used for consumer-loan evaluation. Therefore, the potential presents itself for utilization in the business sector.

Because such important variables as the purpose of the loan, its maturity, the security involved, the deposit status of the applicant, and the particular characteristics of the bank are not explicitly considered in the model, the MDA should probably not be used as the only means of credit evaluation. The discriminant Z score index can be used, however, as a guide in efforts to lower

34. D. D. Hester, "An Empirical Examination of a Commercial Loan Offer Function," *Yale Economic Essays*, vol. 2, No. 1 (1962), pp. 3-57.

35. K. Cohen, T. Gilmore, and F. Singer, "Banks Procedures for Analyzing Business Loan Applications," *Analytical Methods in Banking*, K. Cohen and F. Hammer (eds.) (Homewood, Ill.: Richard D. Irwin, Inc., 1966), pp. 218-251.

the costs of investigation of loan applicants. Less time and effort would be spent on companies whose Z score is very high, i.e., above 3.0, while those with low Z scores would signal a very thorough investigation. This policy would be advisable to the loan officer who had some degree of faith in the discriminant analysis approach, but who did not want his final decision to depend solely on a numerical score. Also, the method would be particularly efficient in the case of short-term loans or relatively small loans where the normal credit evaluation process is very costly relative to the expected income from the loan. Herein lie important advantages of the MDA model—its simplicity and low cost.

Internal Control Considerations and Investment Criteria. An extremely important, but often very difficult, task of corporate management is to periodically assess honestly the firm's present condition. By doing so, important strengths and weaknesses may be recognized and, in the latter case, changes in policies and actions will usually be in order. The suggestion here is that the discriminant model, if used correctly and periodically, has the ability to predict corporate problems early enough so as to enable management to realize the gravity of the situation in time to avoid failure. If failure is unavoidable, the firm's creditors and stockholders may be better off if a merger with a stronger enterprise is negotiated before bankruptcy.

The potentially useful applications of an accurate bankruptcy predictive model are not limited to internal considerations or to credit evaluation purposes. An efficient predictor of financial difficulties could also be a valuable technique for screening out undesirable investments. On the more optimistic side it appears that there are some very real opportunities for benefits. Since the model is basically predictive the analyst can utilize these predictions to recommend appropriate investment policy. For instance, observations suggest that while investors are somewhat capable of anticipating declines in operating results of selective firms, there is an overwhelming tendency to underestimate the financial plight of the companies which eventually go bankrupt. Firms in the original sample whose Z scores were below the so-called "zone of ignorance" experienced an average decline in the market value of their common stock of 45 per cent from the time the model first predicted bankruptcy until the actual failure date (an average period of about 15 months).

While the above results are derived from an admittedly small sample of very special firms, the potential implications are of interest. If an individual already owns stock in a firm whose future appears dismal, according to the model, he should sell in order to avoid further price declines. The sale would prevent further loss and provide capital for alternative investments. A different policy could be adopted by those aggressive investors looking for short-sale opportunities. An investor utilizing this strategy would have realized a 26 per cent gain on those listed securities eligible for short-sales in the original sample of bankrupt firms. In the case of large companies, where bankruptcy occurs less frequently, an index which has the ability to forecast downside movements appears promising. This could be especially helpful in the area of efficient portfolio selection. That is, firms which appear to be strongly susceptible to downturns, according to the discriminant model, would be rejected re-

ardless of any positive potential. Conversely, firms exhibiting these same downside characteristics could be sold short, thereby enabling the portfolio manager to be more aggressive in his other choices.

VI. CONCLUDING REMARKS

This paper seeks to assess the analytical quality of ratio analysis. It has been suggested that traditional ratio analysis is no longer an important analytical technique in the academic environment due to the relatively unsophisticated manner in which it has been presented. In order to assess its potential rigorously, a set of financial ratios was combined in a discriminant analysis approach to the problem of corporate bankruptcy prediction. The theory is that ratios, if analyzed within a multivariate framework, will take on greater statistical significance than the common technique of sequential ratio comparisons. The results are very encouraging.

The discriminant-ratio model proved to be extremely accurate in predicting bankruptcy correctly in 94 per cent of the initial sample with 95 per cent of all firms in the bankrupt and non-bankrupt groups assigned to their actual group classification. Furthermore, the discriminant function was accurate in several secondary samples introduced to test the reliability of the model. Investigation of the individual ratio movements prior to bankruptcy corroborated the model's findings that bankruptcy can be accurately predicted up to two years prior to actual failure with the accuracy diminishing rapidly after the second year. A limitation of the study is that the firms examined were all publicly held manufacturing corporations for which comprehensive financial data were obtainable, including market price quotations. An area for future research, therefore, would be to extend the analysis to relatively smaller asset-sized firms and unincorporated entities where the incidence of business failure is greater than with larger corporations.

Several practical and theoretical applications of the model were suggested. The former include business credit evaluation, internal control procedures, and investment guidelines. Inherent in these applications is the assumption that signs of deterioration, detected by a ratio index, can be observed clearly enough to take profitable action. A potential theoretical area of importance lies in the conceptualization of efficient portfolio selection. One of the current limitations in this area is in a realistic presentation of those securities and the types of investment policies which are necessary to balance the portfolio and avoid downside risk. The ideal approach is to include those securities possessing negative co-variance with other securities in the portfolio. However, these securities are not likely to be easy to locate, if at all. The problem becomes somewhat more soluble if a method is introduced which rejects securities with high downside risk or includes them in a short-selling context. The discriminant-ratio model appears to have the potential to ease this problem. Further investigation, however, is required on this subject.